

# Data Completion Problem : Variational Formulation, Regularization and Discretization

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We deal with the Data Completion Poisson problem, also called the Cauchy's problem. The special point is that: on a portion of the boundary, Neumann and Dirichlet conditions are given, while the complementary part, no conditions, no data are available. We start by explaining how to obtain an appropriate variational formulation of the problem. Its main properties are state and in particular its severe ill posedness is pointed out. A Lavrentiev regularizing process is proposed and analyzed. Next, we show how to use a domain extension to improve the computational performance of the Lavrentiev method. We therefore switch to the finite element approximations, announce the main estimates and provide some comments. This is the most recent part of the research on the subject (20021-2023). To our knowledge this is the first complete convergence analysis of the full discrete-regularized solution of the data completion problem. The proofs are technical and tedious and are not given in the talk (see papers below).

[1] F. Ben Belgacem, V. Girault and F. Jelassi — Analysis of Lavrentiev-Finite Element Methods for Data Completion Problem. *Numerische Mathematik* **139**, 1-25, 2018.

[2] F. Ben Belgacem, V. Girault and F. Jelassi — Full Discretization of Cauchy's Problem by Lavrentiev-Finite Element Method. *SIAM Journal of Numerical Analysis* **60**, pp 558-584, 2022.

[3] F. Ben Belgacem, V. Girault and F. Jelassi — Uniqueness' Failure for the Finite Element Cauchy-Poisson's Problem. *Computers & Mathematics with Applications* **135**, pp 77-92, 2023.

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